MATHEMATICAL METHODS OF EXPERT ESTIMATION ANALYSIS

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Mathematical methods of processing and analyzing the results of the examination, and the received expert estimations are studied. The analysis of expert opinions is carried out by means of the theory of measurements. The use of the results of ranging by means of the factors of rank correlations of Spearman and Candall is considered. The question of processing and the analysis of paired comparison is studied.

The scientific approach to the substantiation of the decisions taken is a guarantee of successes in management if the thorough analysis of particular problems on the basis of objective data with the use of various scientific methods of decision-making was carried out. The decision making person (DMP) uses not only personal knowledge, personal experience and intuition, but also other people's experience and the advice of experts, i.e. the conclusions of subjective character. Expert estimations do not provide the solution to a problem, they represent the information necessary for the substantiation of the decision, for the development of the final decision made by the DMP. Experts, in essence, model the situation connected with a particular problem, the problem itself, synthesize any objects, estimate their characteristics, their interrelation, property, find arguments and proof, helping DMP to make a decision.

The basis of the theory of expert estimations, specifically its part, which is connected with the analysis of the expert opinions expressed in qualitative (instead of in quantitative) manner, is the representative theory of measurements. This theory is connected with the representation of relations between real objects in the form of numerical relations and is one of the components of econometrics.

The results of ranging the data by different experts can be used with the purpose of solving the following problems: 1) finding common ground between the rankings of two experts; 2) finding common ground between any number of ranking attributes; 3) coordinating the opinions of experts in the group containing more than two experts. To solve the first two problems the factors of ranking correlations of Spearman or Candall are used, and to solve the third one we use Candall's factor of concordation. The factor of ranking correlations of Spearman ρ can be used both for strict, and not strict ranging. Strict ranging calculation is carried out according to the formula:

$$\rho = 1 - \frac{6}{m\left(m^2-1\right)}\sum_{i=1}^{m}\left(r_{1i}-r_{2i}\right)^2,$$

where m is the number of elements; r_{1i} is the a rank attributed by the first expert to i^{th} element; r_{2i} is the rank attributed by the second expert to i^{th} element.

The factor of correlation of Spearman can range from -1 up to +1. If the factor of correlation equals 1, the rankings are identical; if $\rho = -1$, the rankings are opposite; if $\rho = 0$ the rankings are linearly independent (uncorrelated).

Let's notice, that the factor of correlation represents a random variable. If $m > 10$ then the distribution of value $T = \rho\sqrt{\frac{m-2}{1-\rho^2}}$ is the distribution of Student with degree of freedom and if $m > 30$ then value ρ has normal distribution within the parameters of ,

$$D(\rho) = \frac{1}{m-1}.$$

The group opinion of experts can be formed only when individual preferences are well-coordinated. For the estimation of the coordination

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of opinions in a group of more than two persons Candall's concordation factor which has a various appearance is used. For strict ranging the concordation factor C_K is found under the formula $C_K = \frac{12S_C}{n^2 m\left(m^2-1\right)}$, where $S_C = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} r_{ij} - \frac{n(m+1)}{2}\right)^2$, n is the number of experts, m is the number of estimated parameters, r_{ij} is the rank of j[th] element appropriated by i[th] expert. If the value of concordation factor C_K is small, it is necessary to change the structure of the expert group. For this purpose it is necessary to break experts into pairs and to find for each pair the factor of pair ranking correlations of Spearman. Then it is necessary to unite the pairs of experts into subgroups according to the values of the factors of pair correlation. At last, for each subgroup it is necessary to define Candall's concordation factor.

For reception and processing of the qualitative expert information by quantitative methods verbal-numerical scales whose structure includes substantially described names of its gradation and corresponding numerical values or ranges of numerical values can be used.

If the number of experts in a group is $n>30$ the distribution of estimation is considered as normal, and therefore L is used with tables of normal distribution. Otherwise confidential probability is found by means of tables of distribution of Students.

The competence of an expert is the degree of their qualification in acertain field of knowledge. The selection of experts to an expert group should be done on the basis of the analysis of the accessible information on vocational training of the candidates, their scientific degree and academic status, their work experience, their general views and erudition, their participation in other examinations and the level of their achievements in professional work. However, it is possible to take advantage of another opportunity to define the degree of competence of the experts. For this purpose control expert appraisal is carried out, with the assumption, that the right answers to the questions are unknown in advance.

Let's consider an example. The calculations are broken into some stages.

1. Four experts who should estimate the problem of the increase of profitability of an enterprise participate in a control expert appraisal of cost price a_2 and expenses a_3.

Table 1

Estimations of experts

The expert	Estimated elements			
	a_1	a_2	a_3	$a_1+a_2+a_3$
1	8	4	2	14
2	10	2	1	13
3	4	8	2	14
4	5	4	3	12

In table 1 the score received by each expert for the estimation of three elements a_1, a_2, a_3 is specified.

2. Then the table of some mark estimations for each expert by the division of each point by the total point for the given expert is made.

Table 2

Mark estimations

The expert	Estimated elements		
	a_1	a_2	a_3
1	0,57	0,29	0,14
2	0,77	0,15	0,08
3	0,29	0,58	0,13
4	0,42	0,33	0,25
Mean scores	0,51	0,34	0,15

In table 2 the average scores received by each of a_1, a_2, a_3 are specified.

3. The weighed sums of relative mark estimations for each expert are calculated:

For the first expert:

$0,57 \cdot 0,51 + 0,29 \cdot 0,34 + 0,14 \cdot 0,15 = 0,41;$

For the second expert:

$0,77 \cdot 0,51 + 0,15 \cdot 0,34 + 0,08 \cdot 0,15 = 0,45;$

For the third expert:

$0,29 \cdot 0,51 + 0,58 \cdot 0,34 + 0,13 \cdot 0,15 = 0,31;$

For the fourth expert:

$0,42 \cdot 0,51 + 0,33 \cdot 0,34 + 0,25 \cdot 0,15 = 0,36.$

4. The sum of the received weighed estimations is calculated:

$0,41 + 0,45 + 0,31 + 0,36 = 1,53.$

5. The factors of expert competence are found:

For the first expert: $\frac{0,41}{1,53} = 0,26;$

For the second expert: $\frac{0,45}{1,53} = 0,30;$

For the third expert: $\frac{0,31}{1,53} = 0,20;$

For the fourth expert: $\frac{0,36}{1,53} = 0,24.$

Average group competence is 0,25. The factors of competence of the 1st and 4th expert are closer to average group competence, therefore it is possible to believe, that the most competent experts are the first and fourth experts.

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