APPLICATION OF MATRIX GAME AND DOMINATION PRINCIPLE IN SOLVING ECONOMIC PROBLEMS

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The problems of applying the matrix game, as well as the links between the solutions of two matrix games are considered. The lines of matrixes satisfy the requirement of domination (received by the author and stated in the work), The solution of two economic problems: the problems of planning the release of collateral production and the problems of crop planning are suggested.

Problem 1. Planning the release of collateral production

Two city factories have received the task to produce children's toys of five kinds, and it is established that the types of toys should be different, the costs and price of all kinds of toys is identical, all toys will be sold in the city. The first factory, producing toys of type a_i, markets in city $P_{ij} \times n$ of toys, and the second factory, producing toys of type b_j sells $(1 - P_{ij})n$ toys (where $i, j = 1,2,3,4,5$; $0 \le P_{ij} \le 1$).

It is supposed, that the income from selling one toy is equal to *1*, the predicted share of selling toys from the first factory is set by matrix $P_{5\times5} = (P_{ij})$:

$$P_{5\times5} = \begin{pmatrix} 0{,}5 & 0{,}5 & 0{,}4 & 0{,}5 & 0{,}2 \\ 0{,}5 & 0{,}4 & 0{,}7 & 0{,}1 & 0{,}6 \\ 0{,}2 & 0{,}3 & 0{,}4 & 0{,}1 & 0{,}7 \\ 0{,}3 & 0{,}6 & 0{,}7 & 0{,}3 & 0{,}2 \\ 0{,}4 & 0{,}4 & 0{,}3 & 0{,}0 & 0{,}2 \end{pmatrix},$$

And all toys (*10 000*) will be sold.

It is required to calculate the release of the types of toys by each factory so that to provide a maximum of profit to each factory.

Solution. It is obvious, that the interests of factories on manufacture and sale of toys are different, each of them aspires to receive maximum profit. We make a theoretic-game model of a disputed situation with the factories (players). The matrix of prizes $H_{5\times5}$ for the first player can be written down to consider the set matrix $P_{5\times5}$ and the income from selling one toy is equal to *1*, and all *10 000* pieces will be sold:

$$H_{5\times5} = \begin{pmatrix} 5000 & 5000 & 4000 & 5000 & 2000 \\ 5000 & 4000 & 7000 & 1000 & 6000 \\ 2000 & 3000 & 4000 & 1000 & 7000 \\ 3000 & 6000 & 7000 & 3000 & 2000 \\ 4000 & 4000 & 3000 & 0 & 2000 \end{pmatrix}.$$

Let's notice, that each element of matrix $H_{5\times5}$ of the first player in situation (i, j) is equal $P_{ij}n$, where $n = 10000$, and P_{ij} - an element of matrix $P_{5\times5}$ $(i, j = 1,2,3,4,5)$.

It is possible to write down the matrix of the second player. The income of the second player in situation (i, j) is equal $(1 - P_{ij})n$, and the sum of incomes of players is equal in each situation (i, j) to the same n: $P_{ij}n + (1 - P_{ij})n = n$.

It specifies the contrast of interests of factories (players) and the presence of the conflict. Matrix *H* models this conflict. Knowing matrix *H*, we shall find optimum strategy x^* and y^* accordingly of the first and second player, a population mean of the first player (value of game) $v_* = H(x^*, y^*)$. Using the results received by the author in item 3.5.9 and 3.5.10 of monography we shall consistently simplify matrix *H* (hence and game with a matrix

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H) . First matrix H we increase the number $k = \frac{1}{1000}$ and receive matrix

$$H^1_{5\times5} = \begin{pmatrix} 5 & 5 & 4 & 3 & 2 \\ 5 & 4 & 7 & 1 & 6 \\ 2 & 3 & 4 & 1 & 7 \\ 3 & 6 & 7 & 3 & 2 \\ 4 & 4 & 3 & 0 & 2 \end{pmatrix}.$$

It is easy to see, that the first vector-line of matrix $H^1_{5\times5}$ dominates the fifth vector-line, i.e. all its elements surpass the corresponding elements of the fifth line. Similarly, the elements of the fourth vector-column do not surpass the corresponding elements of the first and the second vector-column. Therefore the fourth vector-column dominates the first and second vector-columns of the matrix.

Remark. We shall formulate the obvious offers concerning the domination of lines and columns of a matrix.

Offer 1. If i-th vector-line of matrix, the convex linear combination of the others surpass vector-lines with factors *1, 0, 0, ..., 0* the vector-line, entering into this combination with factor *1*, will dominate the specified i-th vector-line.

Offer 2. If j-th vector-column of matrix surpasses a convex linear combination of the other vector-columns of this matrix with factors *1, 0, 0, ..., 0* the vector-column entering into this combination with factor *1*, will dominate j-th vector-column of this matrix.

The remark was necessary for making the basis to the theorems of item 3.5.10 of the author's monograph..

So the first pure strategy dominates the fifth pure strategy, and the fourth pure strategy of the second player dominates its first and second pure strategies.

Consequently, there was an opportunity to reduce studying the properties of game solutions with matrix $H^1_{5\times5}$ to game solutions with matrix

$$H^2_{4\times3} = \begin{pmatrix} 4 & 5 & 2 \\ 7 & 1 & 6 \\ 4 & 1 & 7 \\ 7 & 3 & 2 \end{pmatrix},$$

received from matrix $H^1_{5\times5}$ removing its fifth line, the first and second columns. More precisely, if $x^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ and $y^* = (y_3^*, y_4^*, y_5^*)$ - the optimal mixed strategy of the first and second players, and v_* - value of the game with matrix $H^2_{4\times3}$, game solution with matrix $H^1_{5\times5}$ will be following three of the objects $\{x^* = (x_1^*, x_2^*, x_3^*, x_4^*, 0),$ $y^* = (0, 0, y_3^*, y_4^*, y_5^*), v^*_{H_{4\times3}}\}$.

It is easy to check up, that first vector-column $b_1 = (4,7,4,7)$ of matrix $H^2_{4\times3}$ strictly surpasses a convex linear combination the second and third vector-columns of this matrix with factors $\frac{3}{5}$ and $\frac{2}{5}$:

$$\frac{3}{5}\bar{b}_2 + \frac{2}{5}\bar{b}_3 = \frac{3}{5}\times(5,1,1,3) + \frac{2}{5}\times(2,6,7,2) =$$

$$= \left(\frac{19}{5}, 3, \frac{17}{5}, \frac{13}{5}\right).$$

Having removed from matrix $H^2_{4\times3}$ the first column, we shall receive matrix $H^3_{4\times2} = \begin{pmatrix} 5 & 2 \\ 1 & 6 \\ 1 & 7 \\ 3 & 2 \end{pmatrix}$.

If $\{\tilde{x}^* = (\tilde{x}_1^*, \tilde{x}_2^*, \tilde{x}_3^*, \tilde{x}_4^*), \tilde{y}^* = (\tilde{y}_4^*, \tilde{y}_5^*), \tilde{v}^*_{H^3_{4\times2}}\}$ - game solution with matrix $H^3_{4\times2}$ set, where $\{(\tilde{x}_1^*, \tilde{x}_2^*, \tilde{x}_3^*, \tilde{x}_4^*), (0, \tilde{y}_4^*, \tilde{y}_5^*), \tilde{v}^*_{H^3_{4\times2}}\}$ is game solution with matrix $H^2_{4\times3}$ Analyzing the lines of matrix $H^3_{4\times2}$, we notice that the elements of the second line no more correspond the elements of the third line, and the elements of the fourth line no more correspond the elements of the first line. Hence, the third pure strategy of the first player dominates its second pure strategy, and the first pure strategy dominates the fourth pure strategy. Further, having deleted in matrix $H^3_{4\times 2}$ the second and fourth lines, we shall receive matrix $H^4_{2\times 2} = \begin{pmatrix} 5 & 2 \\ 1 & 7 \end{pmatrix}$ of the second order which has no saddle-element as $v_1 = \max_i \min_j h_{ij} = 2$, and $v_2 = \min_j \max_i h_{ij} = 5$, $v_1 \neq v_2$.

Therefore game with matrix $H^4_{2\times 2}$ is solved in a mixed strategy.

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