MODEL OF SYNTHESIS OF THE MACHINE-BUILDING ENTERPRISE WORKERS MONETARY STIMULATION COORDINATED SYSTEM IN VIEW OF OPERATIONAL LABOUR INPUT

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Keywords: labour input, monetary stimulation, mathematical model, function, function of expenses, the coordination of interests, and the optimum size of surcharges.

The problem of synthesis of machine-building enterprise workers monetary stimulation system depending on labour input performance is considered and the coordinated system of monetary stimulation is directed on the increase of the efficiency of the enterprise.

We shall consider this problem on an example of assembly-conveyor manufacture of joint-stock company "AVTOVAZ". The key feature here is that all labour process is subdivided into a number of fine operations equal or multiple in duration. The coordination of economic interests of the enterprise staff is reached by the means of material stimulation of workers, i.e. the wage fund. The key moments here are the tariff which allows to calculate the parts of the wage and the definitions of the level of surcharges by industrial worker depending on the conditions and results of their work, namely: performance norm tasks, level of labour intensity and labour input. With a view of researching the influence of the rate of surcharges on the performance of certain tasks by an industrial worker we shall consider the model of the system of material stimulation of the workers of assembly-conveyor manufacture depending on labour input.

Workers have different scheduled (technological) labour input, even within the limits of one brigade, therefore we shall consider the construction of a model of mechanisms of stimulation depending on labour input in view of individual worker workload.

According to the analysis of this system the level of performance represents a parity of actual and scheduled volumes of release:

$$\delta_i = \frac{y_i}{x_i}. \qquad (1)$$

where y_i is the actual volume of release calculated per suitable production, per car-complete set; x_i is the scheduled volume of assembly of cars, per car-complete set.

The volume of a brigade output (scheduled and actual) in car-complete sets can be defined from the rate of assembly of cars on the conveyor and the time needed:

$$y_i = a_y \cdot t_p, \quad x_i = a_x \cdot t_p, \qquad (2)$$

where a_y is the actual rate of assembly, piece/hour; a_x is the scheduled output, piece/hour; t_p is the time of needed to complete the work, hour.

The rate of assembly of cars on the conveyor is defined on the basis of the amount of norm-hours planned for the assembly and the actual norm-hours, and also average labour input of one operation of *i*-th worker:

$$a_x = \frac{n_{onep}}{\tau_x}, \quad a_y = \frac{n_{onep}}{\tau_y}, \qquad (3)$$

where n_{onep} is the amount of norm-hours, n/hour; $ф_x$ is the scheduled labour input per one car-complete set, hour; $ф_y$ is the actual labour input per one car-complete set, hour.

The function of stimulation of the industrial worker, defined in view of labour input of assembly of cars is the following:

$$H_i(\tau_{yi}) = T_i\left(1+\left(\frac{\tau_{xi}}{\tau_{yi}K_i}-d\right)\frac{\alpha_i}{1-d}\right) \to \max, \qquad (4)$$

where $ô_{xi}$ is the scheduled labour input per *i*-operation of car assembly by *i*-th worker, hour; $ô_{yi}$ is the labour input per *i*-operation of car assembly by *i*-th worker, hour; K_i is the normative factor of employment of *i*-th worker.

The parity of technological and actual labour input reflects the labour efficiency standard (per-

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formance of a production target) and is derived from the expression:

$$y_{\text{в}i} = \frac{q_i \tau_{xi}}{q_i \tau_{yi} K_i} = \frac{\tau_{xi}}{\tau_{yi} K_i}, \tag{5}$$

where q_i is the scheduled number of assembled cars per certain period of time, pieces; $q_i\hat{\rho}_{xi}$ is the scheduled operating time of i-th worker, hour; $q_i\hat{\rho}_{yi}K_i$ is the actual operating time of i-th worker, hour.

The criterion of the worker's functioning thus looks like the following:

$$f_i(\tau_{yi}) = \left(H_i(\tau_{yi}) - \frac{\gamma}{\tau_{yi}^{2}} \right) t_{\phi i} \to \max_{\tau_{yi}}, \tag{6}$$

where $\tilde{a}$ is the factor of function of expenses of the worker (translates expenses into cost expression).

The factor of the function of expenses is defined as follows:

$$H_i(\tau_{yi}^{пред}) = \frac{\gamma}{(\tau_{yi}^{пред})^2}. \tag{7}$$

According the theory of the organization and industrial management by the marginal level of performance can be 1,3 times exceeding the established norms of the organization of work at the enterprise. According to it the marginal level of actual labour input is defined by the following expression:

$$\delta_i^{пред} = \frac{\tau_{xi}}{K_i \tau_{yi}^{пред}}. \tag{8}$$

According to the equation (7) we shall define the factor of for-expenditure function which translates the efforts of the worker into cost expression:

$$\gamma = H_i \cdot (\tau_{yi}^{пред})^2. \tag{9}$$

Thus we have come to the solution of the task. The function of the worker in view of actual labour input becomes the following:

$$f_i(\tau_{yi}) = \left(T_i + T_i \left(\frac{\tau_{xi}}{K_i \tau_{yi}} - d \right) \frac{\alpha_i}{1-d} - \frac{\gamma}{\tau_{yi}^{2}} \right) t_{\phi i} \to \max_{\tau_{yi}}. \tag{10}$$

The function of administration is the minimization of expenses on stimulation:

$$F(\alpha_i, \tau_{yi}^*) = \sum_{i=1}^{n} H_i t_{\phi i} = \sum T_i \left(1 + \left(\frac{\tau_{xi}}{K_i \tau_{yi}^*} - d \right) \frac{\alpha_i}{1-d} \right) t_{\phi i} \to \min_{\alpha_i}, \tag{11}$$

After the formalization of purposes of the participants of the system we shall state the problem by defining the optimum size of additional payment for the intensity of work of industrial workers in view of the interests of both management and workers:

$$\begin{cases} F(\alpha_i, \tau_{yi}^*) = \\ = \sum_{i=1}^{n} T_i \left(1 + \left(\frac{\tau_{xi}}{K_i \tau_{yi}^*} - d \right) \frac{\alpha_i}{1-d} \right) t_{\phi i} \to \min_{\alpha_i}, \\ T_i \left(1 + \left(\frac{\tau_{xi}}{K_i \tau_{yi}^*} - d \right) \frac{\alpha_i}{1-d} \right) - \\ - \frac{\gamma}{(\tau_{yi}^*)^2} \geq T_i \left(1 + \left(\frac{\tau_{xi}}{K_i \tau_{yi}} - d \right) \frac{\alpha_i}{1-d} \right) - \\ - \frac{\gamma}{(\tau_{yi})^2}, \forall \tau_{yi} > 0, \\ T_i \left(1 + \left(\frac{\tau_{xi}}{K_i \tau_{yi}} - d \right) \frac{\alpha_i}{1-d} \right) \geq R. \end{cases} \tag{12}$$

The mathematical description of this model allows to define the optimum rate of surcharge for performance norm tasks.

An industrial worker carries out operations with certain intensity depending on the set of technological labour input. The purpose of an industrial worker is to maximize the income. Having solved the problem of optimization (12), we shall get the following results:

$$f_i(\tau_{yi}) = T_i + T_i \left(\frac{\tau_{xi}}{K_i \tau_{yi}} - d \right) \frac{\alpha_i}{1-d} - \frac{\gamma}{\tau_{yi}^{2}} \to \max_{\tau_{yi}},$$

$$f_i(\tau_{yi}) = T_i + \frac{\alpha_i\,\tau_{xi}\,T_i}{K_i\tau_{yi}\,(1-d)} - \frac{\alpha_i\,dT_i}{1-d} - \frac{\gamma}{\tau_{yi}^2} \to \max_{\tau_{yi}},$$

$$\frac{\partial f_i}{\partial \tau_{yi}} = 0, \quad \frac{\partial f_i}{\partial \tau_{yi}} = -\frac{\alpha_i\tau_{ix}\,T_i}{\tau_{iy}^2 K_i(1-d)} + \frac{2\gamma}{\tau_{iy}^3} = 0, \quad \frac{\alpha_i\,\tau_{xi}\,T_i}{K_i(1-d)} = \frac{2\gamma}{\tau_{yi}},$$

$$\tau^*{}_{yi}(\alpha_i) = \frac{2\gamma K_i(1-d)}{\alpha_i\tau_{xi}T_i}. \qquad (13)$$

The dependence of labour input on the surcharge for the intensity of work (13) allows to define the optimum size of workers stimulation from the point of view of administration. For this purpose the expression (13) will be substituted into the function of the center (12) and we shall have the solution to the problem of optimization concerning the size of surcharges α.

It is important to note, that the tariff rate for workers of the considered enterprise is higher than the average one in the city. Hence, the second restriction in (12) is carried out at any values of the size of surcharges for performance norm tasks.

$$F = \sum_{i=1}^{n}\left(T_i + T_i\left(\frac{\tau_{xi}}{K_i\tau_{yi}} - d\right)\frac{\alpha_i}{1-d}\right)t_{\phi i} \to \min,$$

$$F = \sum_{i=1}^{n}\left(T_i + \frac{\alpha_i\,\tau_{xi}T_i}{K_i\tau_{yi}(1-d)} - \frac{\alpha_i dT_i}{1-d}\right)t_{\phi i} \to \min,$$

$$F = \sum_{i=1}^{n}\left(T_i + \frac{\alpha_i\,\tau_{xi}T_i}{K_i(1-d)}\cdot\frac{\alpha_i\tau_{xi}T_i}{2\gamma\,K_i(1-d)} - \frac{\alpha_i\,dT_i}{1-d}\right)t_{\phi i} \to \min,$$

$$F = \sum_{i=1}^{n}\left(T_i + \frac{\alpha_i^2\tau_{xi}^2\,T_i^2}{2\gamma\,K_i^2(1-d)^2} - \frac{\alpha_i\,dT_i}{1-d}\right)t_{\phi i} \to \min, \qquad (14)$$

thus

$$\frac{dF}{d\alpha_i} = 0, \frac{dF}{d\alpha_i} = \frac{2\alpha_i\,\tau_{xi}^2T_i^2}{2\gamma\,K_i^2(1-d)^2} - \frac{dT_i}{1-d} = 0$$

$$\frac{\alpha_i\,\tau_{xi}^2\,T_i^2}{\gamma\,K_i^2(1-d)^2} = \frac{dT_i}{1-d}. \qquad (15)$$

As a result the expression for the optimum size of the surcharges for the intensity of work will be the following:

$$\alpha_i{}^* = \frac{d(1-d)\gamma\,K_i^2}{\tau_{xi}^2 T_i}. \qquad (16)$$

As the final result we have received the dependence of the intensity of work of a worker (labour input) on the size of the rate of surcharge for the intensity (15) and the expression for the optimum size of the surcharge for the intensity of work (16).

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Received for publication on 07.08.2009