MATHEMATICAL STATEMENT AND FINDING SOLUTION TO THE PROBLEM OF STIMULATION IN ORGANIZATIONAL SYSTEMS

© 2009 D.Y. Ivanov*

Keywords: organizational system, form, structure, admissible sets, criterion function, knowledge and order of functioning, problem of interests' coordination, coordinated decision.

The description of statement and general approach to finding solutions to the problem of coordinating the economic interests of participants in organizational systems due to use of mathematical models of stimulation is considered. Applications of mathematical models of stimulation allow raising the efficiency of the functioning of the organization.

The basic device for modeling the problems of stimulation in the theory of management is the theory of games which is the section of applied mathematics researching the models of decision-making in the conditions of the conflict of interests of the parties (players) when each party aspires to influence the development of a situation in its own interests. The elementary game model is the interaction of two players. The center and the agent are subordinate to it. Such organizational system has the following structure: on the top level of hierarchy there is the center, on the bottom - the agent subordinate to it. The employer, the direct head of the agent or the organization which signed the contract with the agent can be the center. The employee, the subordinate or the organization, being the second party under the corresponding contract can be the agent.

The strategy of the agent is the choice of the action $y \in A$ belonging to the admissible set (that is, satisfying the existing restrictions) of actions A. The set of admissible actions represents the set of alternatives from which the agent makes the choice, for example, the range of possible duration of working hours, non-negative and not exceeding technological restrictions volume of production, etc.

The rule of accepting the decisions of the center concerning the stimulation of the agent refers to *the mechanism of stimulation*. The mechanism of stimulation includes the system of stimulation which within the limits of the models considered in the present work, is completely defined by the function of stimulation. The function of stimulation sets the dependence of the rate of commission of the agent received by it from the center, defined by the actions chosen by it.

The interests of the participants of the organizational system (the center and the agent) are reflected by their criterion functions which we shall designate accordingly: $\Phi(y)$ and $f(y)$.

Criterion functions represent: for the agent a difference between stimulation and expenses:

$$f(y) = \sigma(y) - c(y),$$

and for the center there is a difference between the income and the expenses of the center for stimulation and the compensation, paid to the agent:

$$\Phi(y) = H(y) - \sigma(y).$$

The rational behavior of the participant of OS consists in the maximization (the choice of own strategy) of its criterion function in view of all the information available and the existing restrictions.

As the value of the criterion function of the agent depends both on its own strategy, and on the function of stimulation within the limits of the accepted hypothesis of rational behavior the agent will choose the actions whose stimulations at the set system maximizes its criterion function. It is clear, that the set of such actions depends on the system of stimulation used by the center. The basic idea of stimulation consists in varying the system of stimulation.

The problem of stimulation consists in finding such system of stimulation which provides maximum efficiency. The set of actions of the agent delivering the maximum of its criterion function (depending on the function of stimulation), is referred to as the set of *game solutions*, or *set of the actions sold by given system of stimulation*:

$$P(\sigma) = \operatorname{Arg} \max_{y \in A} \{\sigma(y) - c(y)\}.$$

* Dmitry Y. Ivanov, PhD in Economics, senior lecturer of Samara State Aerospace University. E-mail: ssau_ivanov@mail.ru.

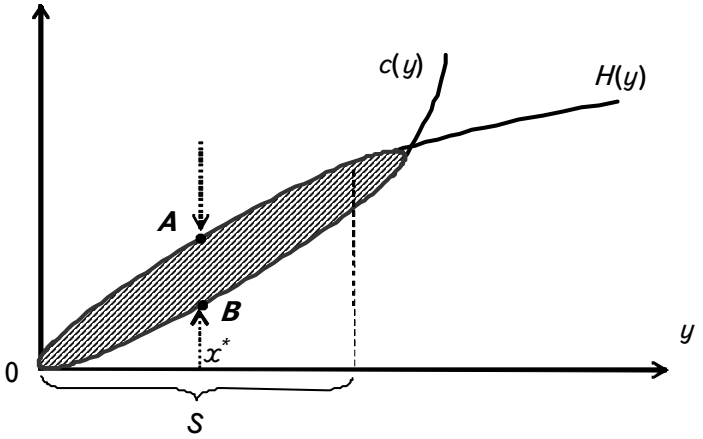

Fig. **The optimum solution to the problem of stimulation**

Hence, the system of the effectiveness of stimulation $\sigma \in M$ equasl:

$$K(\sigma) = \max_{y \in P(\sigma)} \Phi(y),$$

where it is defined (2).

The problem of synthesizing the optimum system of stimulation consists in choosing the admissible system of stimulation with maximum efficiency:

$$K(\sigma) \to \max_{\sigma \in M}.$$

On figure the schedules of the functions $H(y)$ and $c(y)$ are represented. From the point of view of the center the stimulation cannot exceed the income received by it from the activity of the agent (as, having refused interaction with the agent, the center can always receive zero utility). Hence, the admissible decision is below the function $H(y)$. From the point of view of the agent the stimulation cannot be less, than the sum of expenses and reserve utility (which the agent can always receive, choosing zero action). Hence, the admissible decision is above function $c(y)$.

The set of actions of the agent and the corresponding values of the criterion functions satisfying simultaneously all the above-listed restrictions form the *"area of compromises"* shaded in Figure. The set of actions of the agent where the area of the compromise is not empty is referred to as the *set of the coordinated decisions:*

$$S = \{x \in A \mid H(x) \geq c(x) \text{ i } 0\}.$$

The analysis tells us that the solution to the problem of stimulation can be divided into two stages. At the first stage the problem of the coordination is solved - the set of actions sold for the set of restrictions is defined - the set of the coordinated plans. At the second stage the problem of the optimum coordinated planning is solved - search for the sold action which is the most preferable from the point of view of the center.

Gubko M.V., Novikov D.A. Game theory in managing organizational systems. M., 2002.

Myerson R.B. Game theory: analysis of conflict. L., 1991.

Kochieva T.B., Novikov D.A. Basic stimulating systems. M., 2000.

Novikov D.A. Stimulating in organizational systems. M., 2003.

Novikov D.A. Summarized solutions to the problems of stimulating in active systems. M., 1998.

Novikov D.A. Stimulating in social and economic systems (basic mathematical models). M., 1998.

Received for publication on 09.07.2009