PECULIARITIES OF THE SYSTEM OF SIMULTANEOUS EQUATIONS FOR MODELING THE STABILIZATION PROCESSES IN THE RF ECONOMY

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The article suggested econometric model relationship of major socio-economic indicators, describing the stabilization process of the RF economy over the period from January 1998 to November 2008. The model is described by the system of simultaneous equations. With the aid of the given model single-value and interval prediction of indicators of stabilization of the RF economy can be carried out.

Among the reasons hindering the process of socio - economic development of Russia the most important one is instability. Thus, the stabilization of the economy[1] is considered to be the main task of the socio - economic reform of the country.

In our view, the stabilization of the economy should be understood as the process of bringing the economic system to stability, characterized by:

♦ relatively small but steady increase of production;

♦ employment growth, corresponding to the natural growth of population;

♦ stable level of well-being of the population;

♦ virtually unchanged prices;

♦ reducing the budget deficit;

♦ equilibrium of foreign trade operations.

To simulate and forecast the stabilization process of the RF economy we can use a system of simultaneous equations. To construct this system of simultaneous equations (SSE) we will apply two-stage least squares (TSLS).

The initial information base includes monthly research data from January 1998 to November 2008 containing some indicators of the stabilization process, namely: the factors of economic development (including indicators characterizing industry, agriculture, construction, trade, and more etc.), the factors that determine foreign economic relations of Russia and non-economic factors of social development[2].

In the first phase of construction of the SSE the direction of cause-effect relationship of the concerned indicators using the Granger's[3] test was found. The test allowed to mark out a group of interrelated indicators that were considered endogenous variables of time t ($t = \overline{1;131}$):

$Y_t^{(1)}$ - investment in fixed capital, billions rubles;

$Y_t^{(2)}$ - retail trade turnover, billion rubles;

$Y_t^{(3)}$ - the volume of paid services to the population, billion rubles;

$Y_t^{(4)}$ - the average monthly wage of a worker (nominal) rubles.

According to the results of the Granger's test the following indicators related to the selected endogenous variables were identified:

$X_t^{(1)}$ - usage of space by organizations of all forms of ownership, one million square meters of total area;

$X_t^{(2)}$ - exports of goods, billion U.S. dollars;

$X_t^{(3)}$ - imports of goods, billion U.S. dollars;

$X_t^{(4)}$ - the official exchange rate, rubles per U.S. $ 1;

$X_{t-1}^{(5)}$ - fixed monthly pension a month ago, the average amount, rubles;

$X_{t-3}^{(5)}$ - fixed monthly pension three months ago, the average amount, rubles;

$X_t^{(6)}$ - the total number of unemployed people, millions of people.

Seven of the listed factors were considered an exogenous variables model.

After analyzing the interrelations of endogenous, exogenous and lags exogenous variables the following structural form of the SSE[4] have been received:

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$$\begin{cases} Y_t^{(1)} = \alpha_1 + \beta_{14}Y_t^{(4)} + \gamma_{11}X_t^{(1)} + \gamma_{13}X_t^{(3)} + \gamma_{14}X_t^{(4)} + \gamma_{16}X_t^{(6)} + \varepsilon_t^{(1)} \\ Y_t^{(2)} = \alpha_2 + \beta_{23}Y_t^{(3)} + \gamma_{21}X_t^{(1)} + \gamma_{24}X_t^{(4)} + \varepsilon_t^{(2)} \\ Y_t^{(3)} = \alpha_3 + \beta_{31}Y_t^{(1)} + \beta_{32}Y_t^{(2)} + \gamma_{32}X_t^{(2)} + \gamma_{33}X_t^{(3)} + \gamma_{35}X_{t-1}^{(5)} + \varepsilon_t^{(3)} \\ Y_t^{(4)} = \alpha_4 + \beta_{43}Y_t^{(3)} + \gamma_{41}X_t^{(1)} + \gamma_{45}X_{t-3}^{(5)} + \varepsilon_t^{(4)} \end{cases} \quad (1)$$

Where $\varepsilon_t^{(i)}$ indicates random components; α_i, β_{ik}, γ_{ip} indicate structural coefficients ($i = \overline{1;4}$, k=$\overline{1;4}$, $p = \overline{1;7}$).

The verification of conditions identifiability[5], carried out beforehand, showed that all the equations of SSE's were over-identified. This means that for every equation the number of exogenous variables excluded from the equation is strictly greater than the number of included endogenous variables minus unit (for the first equation 3>(2-1), for the second - 5>(2-1), for the third - 4>(3-1), for the fourth - 5>(2-1)). Therefore, to estimate the structural parameters of the model two-stage least squares method was applied.

The first step of two-stage least squares is the building of regression equation $Y_t^{(4)}$ for all p of explanatory variables with the help of usual ordinary least squares.

After the construction of principal components of F_1, F_2 and F_3 for $Y_t^{(4)}$ the most informative should be selected. As a criterion of informativity[6] the maximum of the lower confidence countour coefficient of determination $R^2_{\min}$, calculated as follows, was used:

$$R^2_{\min} = \hat{R}^2 - 2 \cdot \sqrt{\frac{2p(n-p-1)}{(n-1)(n^2-1)}}\left(1 - R^2\right),$$

Where R^2 is the coefficient of determination, which determines the degree of connection between the endogenous variable on the one hand and a set of exogenous variables and principal components - on the other, $\hat{R}^2$ is the corresponding adjusted coefficient of determination, n is the number of observations, p is the number of explanatory variables.

All possible variants for a set of predetermined variables for the calculation $Y_t^{(4)}$ are presented in table 1.

Then regression of endogenous variable $Y_t^{(4)}$ was built by variables $X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}$, F_1, F_2. Next two-stage least square was conducted against the usual pattern. As a result, the following equation was received:

$$Y_t^{(1)} = -91{,}22 - 0{,}01Y_t^{(4)} + 12{,}77X_t^{(1)} + 33{,}4X_t^{(3)} + 5{,}07X_t^{(4)} - 11{,}72X_t^{(6)} \quad (2.1)$$

$t_{observ.period}$(-2.24) (-2.38) (10.52) (7.21) (4.69) (-2.52)
R^2= 0,938.

Table 1

A set of predetermined variables for the calculation $Y_t^{(4)}$ in the construction of the first equations of the system

Variables	Value $R^2_{\min}$
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_1$	0,99069
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_2$	0,971034
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_3$	0,964726
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_1, F_2$	0,992647
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_1, F_3$	0,990883
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_2, F_3$	0,971038
$X_t^{(1)}, X_t^{(3)}, X_t^{(4)}, X_t^{(6)}, F_1, F_2, F_3$	0,992638

This equation can be interpreted as follows: investment in fixed assets is most sensitive to the changes in imports in the current period ($X_t^{(3)}$) and least sensitive to the changes in the average monthly wage of a worker (nominal) in the current period ($Y_t^{(4)}$).

In each of the three remaining equations of the system the procedure of finding of approximation function $Y_t^{(i)}$ ($i = \overline{2;4}$) was carried out in a similar way.

After using two-stage least square for assess the structural parameters of the remaining equations the following result was received:

$$\underset{t_{\text{íàáë}}}{Y_t^{(2)}} = \underset{(-2{,}22)}{-18{,}62} + \underset{(120{,}75)}{2{,}83Y_t^{(3)}} + \underset{(10{,}63)}{5{,}88X_t^{(1)}} + \underset{(5{,}87)}{1{,}85X_t^{(4)}} \qquad (2.2)$$

$$R^2 = 0{,}994,$$

$$\underset{t_{\text{íàáë}}}{Y_t^{(3)}} = \underset{(-21{,}33)}{-20{,}46} - \underset{(-5{,}52)}{0{,}08Y_t^{(1)}} + \underset{(6{,}12)}{0{,}14Y_t^{(2)}} + \underset{(7{,}48)}{2{,}18X_t^{(2)}} + \underset{(7{,}27)}{3{,}24X_t^{(3)}} + \underset{(7{,}32)}{0{,}03X_{t-1}^{(5)}} \qquad (2.3)$$

$$R^2 = 0{,}996$$

$$\underset{t_{\text{íàáë}}}{Y_t^{(4)}} = \underset{(-4{,}39)}{-309{,}45} + \underset{(13{,}96)}{37{,}95Y_t^{(3)}} + \underset{(7{,}96)}{78{,}37X_t^{(1)}} + \underset{(2{,}94)}{0{,}72X_{t-3}^{(5)}} \qquad (2.4)$$

$$R^2 = 0{,}99.$$

To obtain the predictive values of endogenous variables its reduced form was constructed:

$$Y_t^{(1)} = 12{,}08X_t^{(1)} - 1{,}44X_t^{(2)} + 33{,}02X_t^{(3)} + 5{,}17X_t^{(4)} - 0{,}01X_{t-3}^{(5)} - 0{,}02X_{t-1}^{(5)} - 12{,}34X_t^{(6)} - 77{,}53$$

$$Y_t^{(2)} = 5{,}21X_t^{(1)} + 10{,}76X_t^{(2)} + 2{,}8X_t^{(3)} + 1{,}13X_t^{(4)} + 0{,}003X_{t-3}^{(5)} + 0{,}15X_{t-1}^{(5)} + 4{,}63X_t^{(6)} - 97{,}67$$

$$Y_t^{(3)} = -0{,}24X_t^{(1)} + 3{,}8X_t^{(2)} + 0{,}99X_t^{(3)} - 0{,}26X_t^{(4)} + 0{,}001X_{t-3}^{(5)} + 0{,}05X_{t-1}^{(5)} + 1{,}64X_t^{(6)} - 27{,}93$$

$$Y_t^{(4)} = 69{,}39X_t^{(1)} + 144{,}27X_t^{(2)} + 37{,}59X_t^{(3)} - 9{,}7X_t^{(4)} + 0{,}76X_{t-3}^{(5)} + 1{,}99X_{t-1}^{(5)} + 62{,}05X_t^{(6)} - 1369{,}4$$

With the help of the reduced form of the constructed econometric model we were able to make a point prediction of endogenous variables for December 2008 according to the data for exogenous variables for this month. The results are presented in table 2.

It is also possible to built joint confidence interval $1-\alpha$ $(\alpha = 0{,}05)$ for all endogenous variables simultaneously[7]:

$$\hat{y}_{t+\tau}^{(i)} - \sqrt{c_\alpha \hat{s}_{ii}} \le y_{t+\tau}^{(i)} \le \hat{y}_{t+\tau}^{(i)} + \sqrt{c_\alpha \hat{s}_{ii}} \quad ,$$

Where $c_\alpha = \left[1 + X_{t+\tau}^{T}(X^T X)^{-1} X_{t+\tau}\right] \dfrac{(n-p)m}{n-p-m+1}$,

$F_\alpha(m;\ n-p-m+1)$, $F_\alpha(m;\ n-p-m+1) = 100\%$ - percent point, F = distribution with the number of degrees of freedom, equal to m and (n-p-m +1), $\hat{s}_{ii}$ = I - j diagonal matrix element, $\hat{S}_\varepsilon$ = unbiased evaluation of the covariance matrix remainders reduced form

$$\hat{S}_\varepsilon = \frac{1}{n-p}(Y - X\hat{\Pi}_{OLS}^T)^{\mathrm{T}}(Y - X\hat{\Pi}_{OLS}^T),$$

Y = matrix dimension 131×4 of the actual values of endogenous variables, X = matrix di-

Table 2

The point prediction of endogenous variables for December 2008

	$Y_{132}^{(1)}$	$Y_{132}^{(2)}$	$Y_{132}^{(3)}$	$Y_{132}^{(4)}$
Actual values*	1329,8	1453,6	463,2	20238
Predicted values	1347,9	1468,9	471	20015
Relative forecasting error (%)	1,3	1	1,7	1,2

* The preliminary data from a site http: / / www.gks.ru

mension 131×8 of actual values of exogenous variables, $\widehat{\Pi}_{OLS}$ = matrix dimension 4×8 ordinary least squares of estimated coefficients of reduced form a system of simultaneous equations,

$$\widehat{\Pi}_{OLS} = Y^T X (X^T X)^{-1}.$$

For this system of simultaneous equations the matrix $\widehat{S}_{\varepsilon}$ comes in the following form:

$$\begin{pmatrix} 1196{,}51 & 165{,}32 & 18{,}35 & -2083{,}92 \\ 165{,}32 & 205{,}39 & 14{,}36 & 1736{,}83 \\ 18{,}35 & 14{,}36 & 18{,}11 & 339{,}62 \\ -2083{,}92 & 1736{,}83 & 339{,}62 & 81733{,}62 \end{pmatrix}$$

For the endogenous variable $Y_t^{(1)}$, which represents investment in fixed assets, the following inequality was received:

$$1222{,}59 \le Y_{132}^{(1)} \le 1473{,}22.$$

Thus, in December 2008, with the projected values of exogenous variables, the investments in fixed assets (with the probability of 95%) could be in the range between 1222,59 to 1473,22 billion rubles.

Similarly, confidence intervals were obtained for the remaining endogenous variables:

for $Y_t^{(2)}$ = turnover of retail trade $1416{,}9 \le Y_{132}^{(2)} \le 1520{,}82$,

for $Y_t^{(3)}$ = the volume of paid services to the population $455{,}58 \le Y_{132}^{(3)} \le 486{,}4$,

for $Y_t^{(4)}$ = the average monthly wage of a worker (nominal) $18971{,}27 \le Y_{132}^{(4)} \le 21042{,}7$.

Note that the actual values of all the variables also come in corresponding confidence interval (see table 2).

Applying the approach of the simultaneous equations system made it possible to obtain a more accurate model which can be used for forecasting theindicators of economic stabilization in Russia.

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5. See: Aivazian, S.A. Applied Statistics. Fundamentals of Econometrics: Textbook for universities: Vol.2 - T.2: Fundamentals Econometrics. - Moscow: UNITY - Dana, 2001.

6. See: Ibid. P. 85

7. See: Ibid. P. 370

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