THEORETICAL SUBSTANTIATION AND INTERPRETATION OF ESTIMATIONS OF INFLUENCE OF FACTORS IN ECONOMETRICAL MODELING

Keywords: growth rate, moment rate of accession, elasticity coefficient, regression model, partial coefficient of determination.

The article considers a method of adequate valuation of moment and interval rates of growth of occurrence, econometrical approach in research of factor's influence to the dynamic of result value.

Growth rates are the most important statistical indexes, characterized the intensity of development of occurrence. Traditionally it is calculated as correlation of its sizes for equal or through equal intervals. Lack of methods of continuous estimation of intensity of changing of occurrences in theory and practice of econometrical modeling leads to the problem of control and regulation of intensity of development of social-economic processes and accordingly "approximation" of this calculation with an uncertain fault.

Generally it is characterized by the intensity of development of occurrence in point by correlation of derived function to value $\left(\frac{y'}{y}\right)$,

$\frac{y'}{y} = \frac{1}{x}$

calling it as you want (both growth rate or accession rate).

However, we can exemplify variety functions, for which $\frac{y'}{y}\langle -1\ (-100\%)$ in certain interval or in all sphere of definition of function. There is the next functions: $y = c \cdot e^{-rx}$, where is x-argument, - constant values, which are unequal to 0 and ; $y = c \cdot x^{-r}$ where is $0 < x < r$.

For example, for function $y = e^{-7x}$ in every point of sphere its definition $\frac{y'}{y} = -7\ (-700\%)$, that does not corresponds to growth rate or accession rate.

In our opinion, "growth rate" is not suitable for an index $\frac{y'}{y}$, although relating to price index it is called as a "inflation rate", since its value is negative for every decreasing positive on certain interval of function, but growth rates of every occurrence, as it is known, are not negative rates. It is clear that growth rate of occurrences, the levels of which increase more than 1 (100%). However, value $\frac{y'}{y}$ for many increasing functions is less 1, for example, for exponential function ($y = x^n$), for $n = 1$ $y = x$,

where from follows that $\frac{y'}{y} < 1$ where is $x > 1$; for $n = 2$ $y = x^2$, $\frac{y'}{y} = \frac{2}{x}$ и $\frac{y'}{y} < 1$ where is $x > 2$ etc., that contradicts to growth rate, presented in econometrical analysis.

Value of moment growth rate of this function in every point X must be equal . According to the general method of calculation the moment rate of growth of exponential function in every point of sphere of its definition will be equal :

$$\frac{y'}{y} + 1 = \frac{c \cdot a^x \cdot \ln a}{c \cdot a^x} + 1 = \ln a + 1,$$

That contradicts reality.

Make certain that initial preconditions of possible chance of index $\frac{y'}{y}$ as a momentary rate of accession of function is not available.

Evidently,

$$\frac{y'}{y} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{f(x) \cdot \Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\frac{f(x+\Delta x)}{f(x)} - 1\right),$$

* Vladimir A. Prokofyev, Doctor of Economics, Professor, Deputy Head of Statistics department, Saratov State Social and Economic University.

Where is $\frac{f(x+\Delta x)}{f(x)}$ - growth rate of function $y=f(x)$ for interval Δx, - rate of accession of function for interval Δx,

- correlation of basic rate of accession of function for interval Δx to length of this interval.

But it is proved that average rate of accession can‘t be defined by this method, but it is calculated as margin of average geometrical catenarian growth rates and unit in every textbook of statistical theory.

Final confirmation of discordance of index

to appointment from a perspective of social-economic interpretation can be shown by the next example. We suggest that specific gravity of families, having more 2 children in region on 1, January - period is divided on 4 times per 5 years (%):

1993 - 40,0; 1998 - 10,0; 2003 - 4,4; 2008 - 2,5.

It is easy to know that these values are a values of function $y=\frac{10}{x^2}$, where there is an argument x is measured in decades, but conditioned start of count is a 1, January 1988. Then,

As we see that the value of index $\frac{y'}{y}$ in every point of interval $0 < x < 2$ is less than -1 (-100%), that is does not correspond to rate of accession of investigated occurrence, which can‘t decrease more than by 100%.

It is made clear in conditions of this example the reason of fault, when we talk about index $\frac{y'}{y}$ as a moment rate of accession. It is considered that the values of index $\frac{1}{\Delta x}\left(\frac{f(x+\Delta x)}{f(x)}-1\right)$ for interval of decreasing duration [,], where $x=1$, is varied from 1 to 0.

Suggesting that , we can get interval rate of accession of investigated occurrence for 10 years: $\frac{1}{1}\left(\frac{2,5}{10,0}-1\right)=-0,75$; where $\Delta x=\frac{1}{2}$ we can get $\frac{1}{1/2}\left(\frac{4,4}{10,0}-1\right)=2(0,44-1)=$ $=2(-0,56)=-0,56-0,56=-1,12$; where 0,44 is an interval rate of accession of occurrence for 5 years; -0,56 is a interval rate of accession for 5 years, but -1,12 is a double value for the 10 years, not having meaning because wrongful doubling of 5-years rates of accession with purpose getting rate of accession for 10 years.

Evidently, value of index $\frac{1}{\Delta x}\left(\frac{f(x+\Delta x)}{f(x)}-1\right)$ will be run to value $\frac{y'}{y}$ in point $x=1$ in conditions of decreasing length of interval, for which it is measured. So, if $\Delta x=\frac{1}{10}$, we can have:

$$\frac{1}{1/10}\left(\frac{10}{1,1^2}\Big/\frac{10}{1}-1\right)=10(0,83-1)=$$

$$=10(-0,17)=-1,70;$$

where -0,17 is a rate of accession for 1 year, but -1,70 is a sum of equal annual rates of accession for 10 years;

if $\Delta x=\frac{1}{100}$ we can have

$$\frac{1}{1/100}\left(\frac{10}{1,01^2\cdot 10}-1\right)=100(0,9803-1)=$$

$$=100(-0,0197)=-1,97,$$

where -0,0197 is a rate of accession for 10-th part of year.

For definition of real value of growth rate and rate of accession of function in point $x=1$ will discuss by next. Whereas the intensity of development of investigated occurrence for interval [1, $1+\Delta x$], duration of which is a 5 years, is characterized by growth rate, which is equal 0,44. In general case for characteristic of intensity of changing of function for interval Δx in calculation for unit of measurement of argument we need to calculate a value:

$$\bar{i} = \left(\frac{f(x+\Delta x)}{f(x)}\right)^{\frac{1}{\Delta x}}. \quad (1)$$

For estimation of intensity of development of occurrence in point x it is necessary to go to the limit of statement (1) if $\Delta x \to 0$, which we will call as a moment growth rate of function $y = f(x)$ in point x and to specify as :

$$f^T(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x)}{f(x)}\right)^{\frac{1}{\Delta x}}. \quad (2)$$

In result of transformations of expression (2) we can get a convenient for practical using the formula of calculation of moment growth rates.

It is known that $f'(x) = \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$, where from $f(x+\Delta x) = f(x) + [f'(x) + \varepsilon(\Delta x)]\Delta x$, where $\varepsilon(\Delta x) \to 0$ if $\Delta x \to 0$. Then

$$f^T(x) = \lim_{\Delta x \to 0}\left[\frac{f(x) + [f'(x) + \varepsilon(\Delta x)]\Delta x}{f(x)}\right]^{\frac{1}{\Delta x}} =$$

$$= e^{\frac{f'(x)}{f(x)}}. \quad (3)$$

Using formula (3) we define real value of growth rate and rate of accession of function $y = \frac{10}{x^2}$ in point $x = 1$ in conditions of considered above sample. Since correlation $\frac{y'}{y}$ in this point is equal (-2), that growth rate consists: $y^T = e^{-2} = 0{,}135\ (13{,}5\%)$, but rate of accession of share of families with 2 and more children on 1, January 1998 in calculation for 10 years consists -86,5% in region.

Formula (3) establishes the limit connection between absolute accession of function in point x $(f'(x))$, by value of function in this point and its growth rate : $f'(x) = f(x)$.

; $f(x) = \frac{f'(x)}{\ln f^T(x)}$.

In economic, statistical and mathematical literature of science and study the appointment was spread over the next 2 variants of calculation of elasticity of function $y = f(x)$:

- Correlation of interval rates of accession of function (y) and argument (x), calculated for all interval of changing argument (Δx):

$$Э_x(Y) = \left[\frac{y(x+\Delta x)}{y(x)} - 1\right] \Big/ \left[\frac{x+\Delta x}{x} - 1\right] =$$

$$= \frac{\Delta y}{y} \Big/ \frac{\Delta x}{x}; \quad (4)$$

♦Limit of right part of equality (4) if $\Delta x \to 0$:

$$Э_x(Y) = \lim_{\Delta x \to 0}\left(\frac{\Delta y}{y} \div \frac{\Delta x}{x}\right) =$$

$$= \frac{x}{y}\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = x \cdot \frac{y'}{y}. \quad (5)$$

The elasticity of function $y = f(x)$ in accordance with general term of elasticity must answer the question: how much percent of accession of function $y(x)$ is needed in average by 1% of accession of argument x, and therefore must be defined according to correlation of rates of accession of function and argument: or interval according to formula (4), or average, calculated according to unit of estimation of argument for interval in accordance to formula (1):

$$Э_x(Y) = \left[\left(\frac{y(x+\Delta x)}{y(x)}\right)^{\frac{1}{\Delta x}} - 1\right] \Big/ \left[\left(\frac{x+\Delta x}{x}\right)^{\frac{1}{\Delta x}} - 1\right]; \quad (6)$$

Or moment in point x, defined in calculation for unit of estimation of argument according to formulas (2), (3):

$$Э_x(Y) = \left[\lim_{\Delta x \to 0}\left(\frac{y(x+\Delta x)}{y(x)}\right)^{\frac{1}{\Delta x}} - 1\right] \Big/$$

$$\Big/ \left[\lim_{\Delta x \to 0}\left(\frac{x+\Delta x}{x}\right)^{\frac{1}{\Delta x}} - 1\right] =$$

$$= (e^{y'/y} - 1)/(e^{1/x} - 1) = (y^T - 1)/(x^T - 1). \quad (7)$$

First method of calculation of average rate of accession is a known in theory of statistics. Second method is without economic treatment and therefore is not used in science and practical sphere of activity, i.e. a growth of every next level of occurrence by the same number of percent suggests variable, but not constant base of comparison. But this method became in the base of formula (5) of limit transition from average speed of changing of function to its derivative, relation of which to function was called as a "rate of its changing".

Beside formulas (4), (5) there is used their modifications also in statistical literature:

$$Э_x = \frac{\Delta y}{\overline{y}} \Big/ \frac{\Delta x}{\overline{x}}, \qquad (8)$$

$$Э_x = y' \frac{\overline{x}}{\overline{y}}, \qquad (9)$$

Which is cause misunderstanding in treatment of economic content of rate of accession, showing correlation of absolute accession of level to average value of 2 contiguous level.

There is very often for estimation of degree of this influence, when we construct the linear equations of plural regression:

In natural form

$$y = a + b_1 x_1 + b_2 x_2 + ... + b_k x_k . \qquad (10)$$

And in standard scale

$$t_y = \beta_1 t_1 + \beta_2 t_2 + ... + \beta_k t_k , \qquad (11)$$

where $t_y = \frac{y - \overline{y}}{\sigma_y}$, $t_i = \frac{x_i - \overline{x}_i}{\sigma_i}$, $(i = \overline{1,k})$,

$$\beta_i = b_i \frac{\sigma_i}{\sigma_y}; \; b_i = \beta_i \frac{\sigma_y}{\sigma_i}. \qquad (12,\ 13)$$

Are equal value and represented as β-coefficients and average coefficients of elasticity

(14)

or $Э_{yx_i} = \beta_i \frac{\upsilon_y}{\upsilon_{x_i}}$, (15)

where υ_y, υ_{x_i} - coefficients of variation.

Suggesting change of factor by Δx_i from its deviation from average $[\Delta x_i + (x_i - \overline{x}_i)]$ we can get the change t_y by value $\frac{\Delta y}{\sigma_y} = \beta_i \frac{\Delta x_i}{\sigma_i}$. Where from,

$$\beta_i = \frac{\Delta y}{\sigma_y} \Big/ \frac{\Delta x_i}{\sigma_i}. \qquad (16)$$

To show by how much percent the deviation $(y - \overline{y})$ is changed in relation to average quadratic deviation σ_y, if deviation $(x_i - \overline{x}_i)$ will increase by 1% in relation to average quadratic deviation σ_i. Then β_i we can treat as elasticity of variation y for variation and we can use for comparison of influence of variation of different factors to variation of result index.

A coefficient of elasticity can be modify to next:

$$Э_{yx_i} = \left(\frac{\Delta y}{\sigma_y} \Big/ \frac{\Delta x_i}{\sigma_i} \right) \frac{\upsilon_y}{\upsilon_{x_i}} = \frac{\Delta y}{\overline{y}} \Big/ \frac{\Delta x_i}{\overline{x}_i}, \qquad (17)$$

where Δx_i, Δy - are the values of change of variables and y, which are correspond to linear kind of regression.

In case of non- linear regression the formula (17) will the same, but for different intervals of same value from sphere of existing of values x_i will be a different values. Beside, the next construction is more feasible in compare to formula (8):

$$Э_{yx_i} = \frac{\Delta y}{y} \Big/ \frac{\Delta x_i}{x_i}, \qquad (18)$$

where y and - are the current values of result index and factor feature, which previous to their changing by values Δx_i and Δy.

Thus, the indexes (12) and (17), joined by correlation (15), are a different content. Formula (15) shows that transition from estimation of elasticity of function for certain variable to estimation of elasticity of variation of function for variation of variable, can provide, dividing to correlation of coefficients of variation y and .

Beside considering above coefficients of elasticity in comparative analysis of econometrical modeling is used: partial coefficient of determination

$$d_{x_i} = r_{yx_i} \cdot \beta_i . \qquad (19)$$

Which must show by how much percent the variation y is explained by variation and Q-coefficient

$$Q_{x_i} = Э_{yx_i} \cdot \upsilon_{x_i}, \qquad (20)$$

Which not have good interpretation.

In conditions of 2-factor model of linear regression we can show that

$$b_2 = \frac{\sigma^2_{x_1} \cdot \sigma_{x_2 y} - \sigma_{x_1 y} \cdot \sigma_{x_1 x_2}}{\sigma^2_{x_1} \cdot \sigma^2_{x_2} - \sigma^2_{x_1 x_2}},$$

where $\sigma_{x_i y}$ - covariation x_i, y; - covariation x_i, x_j (i, j = 1, 2), $i \neq j$, where from

$b_2 = \frac{\sigma_{x_2 y}}{\sigma^2_{x_2}}$ if x_1 and x_2 are independent, i.e. $\sigma_{x_1 x_2} = 0$.

Then $r_{yx_2} = b_2 \cdot \frac{\sigma_{x_2}}{\sigma_y} = \beta_2$ and d_{x_i} will have clearness of coefficient of determination ($d_{x_i} = r^2_{yx_i}$).

$\sigma_{x_i x_j}$

In opposite case when x_1 and x_2 are dependent, the interpretation d_{x_i} as coefficient of determination is became complicated.

If we say about Q-coefficient, that considering a treatment of multiplicative efficient, we can note

$$Q = b_i \cdot \frac{\bar{x}_i}{\bar{y}} \cdot \frac{\sigma_i}{\bar{x}_i} \text{ or}$$

$$Q = \beta_i \frac{\sigma_y}{\sigma_i} \cdot \frac{\sigma_i}{\bar{y}} = \left(\frac{\Delta y}{\sigma_y} \Big/ \frac{\Delta x_i}{\sigma_i} \right) \cdot \frac{\sigma_y}{\bar{y}},$$

$$Q = \frac{\Delta y}{\bar{y}} \Big/ \frac{\Delta x_i}{\sigma_i}, \qquad (21)$$

Where Δx_i, Δy have the same treatment that in considering above article.

Thus, treatment of -coefficient consist of: by how much percent is changed result index y in relation to average level , if derivative $(x_i - \bar{x}_i)$ is increased by 1% in relation to average quadratic deviation σ_i. This understanding of Q-coefficient is as incorrect interjection $Э_{yx_i}$ and β_i: numerator of formula (21) belongs to elasticity of result index (first part of treatment of $Э_{yx_i}$), but denominator belongs to elasticity y for variation .

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