ECONOMICAL-MATHEMATICAL MODEL OF CARGO SYSTEM FUNCTIONING ADAPTIVE MANAGEMENT

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This work contains an optimal control methodology based on adaptive control of cargo systems, including different economical and technological criteria.

One of the main problem solved by various cargo systems from terminal station and cargo area to railway and the whole transport network is to achieve optimal functional rate. Therefore in this article general principles of control system modeling of cargo system will be reviewed.

To create optimal conditions of every cargo system functioning it is necessary to develop the methodology of cargo system managing. Such methodology can be created on the basis of behavior studying of logical dynamic system which represents synthesis of dynamic system and intellectual regulator that can be developed on the basis of artificial neural network.

Any cargo point can be chosen as a dynamic system. In this article the common issue is reviewed: cargo point has a buffer of certain capacity.

Cargo point functioning can be modeled using inventory theory as well as theory of optimal dynamic system controlling.

Economic indexes of cargo point functioning in the context of this model have to be set appropriate by the total amount of expenses which can be divided into constant and variable ones. The quantity of variable expenses depends on the size of portion which is served by a given cargo point.

Hence, general expenses expression can be presented as

$$C(q) = \frac{C_{const}}{q} + f(q) \times q, \qquad (1)$$

where $C(q)$ - total expenses on freight flow serving; C_{const} - constant expenses for certain cargo point (monetary unit); q - size of consignment which is served by cargo point;   - function which represents the way of dependence of variable expenses on the size of consignment of freight flow.

Optimal size of consignment recommended for this cargo point can be calculated based upon the condition of total expenses minimization:

$$\frac{dC}{dq} = 0. \qquad (2)$$

However, the problem of freight flow parting can be solved only taking into account dynamic freight flow analysis at the input and at the output of cargo system.

Freight flow at the input to the buffer (or directly to the cargo point) can be presented as a differential equation of form:

$$\frac{dQ(t)}{dt} = A(t)Q(t) + B(t)G(t) + \xi(t). \qquad (3)$$

Here $Q(t)$- freight flow throughout cargo point; $\xi(t)$ - random process, characterized by its own average of distribution, set of moments;   - coefficient (in general case - matrix), considering the dynamics of functioning of the cargo system in time; $B(t)$ - coefficient (or matrix) considering history of incoming influences $G(t)$.

One of the standards of correspondence of cargo system functioning to the given parameters is the determination of freight flow expenses C in the specified interval. Consequently cargo system management criterion can be assigned as:

$$F = (C(t) - C_{fix})^2 \to \min, \qquad (4)$$

where F - management quality criterion; C_{fix} - assigned (desired) value of freight flow expenses.

Generalized equation of developing a cargo point as a dynamic system can be set as:

$$\frac{dQ(t)}{dt} = A(t)Q(t) + B(t)G(t) + \xi(t) + u(Q(t), F). \qquad (5)$$

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* Aleksey M. Olshanskiy, Samara State University of Communications.

Last term of the equation in formula (5) represents control action on cargo point depending on system condition and desired criterion correspondence (4).

In general case functioning of any cargo point can be described by irregular linear differential equation. The solution will be compared to two parameters:

1. Desired criterion value (4).
2. Calculated optimal value of freight flow $Q(t)$.

Having analyzed system functioning, neural network is supposed to make a decision:

- "not to interfere in process, it is close to optimal ";
- "process proceeds in permissible limits, interference is not required";
- "slight management correction is necessary " (automatic regulation plan is to be introduced);
- "urgent external actions are necessary" (up to cargo point interruption).

Management in this system can be understood as a simple formal mathematical expression which is necessary to convert into complex operations by cargo system functioning normalization or it is possible to interpret it as a set of certain operations. In that case it is necessary to use a neural network for controlling cargo system functioning. It has to perform pattern recognition task.

Cargo system functioning quality indexes can be introduced in the following way:

x_1 - cargo system functioning delay, hour;

x_2 - amount of delayed freight flow portion, ton;

x_3 - free float of cargo system conversion ability, %;

x_4 - instant unplanned expenses, rubles;

x_5 - operational coefficient of irregularity, %;

x_6 - freight flow processing rate, ton/unit time;

x_7 - loss and distortion percentage, %.

It is recommended to introduce two neuron layers for neural network functioning considering physical and economical sense of the criteria referred above. Entries of the first layer, consisting of three independent neurons each emitting its own output signal, receive information. First layer neuron emits its output signal into the input of the next neuron, which converts information and produces eigenvalue which determines the diagnosis for the cargo system and therefore necessary control action selection. Parameters referred above form a group by neurons in the following way:

- first layer first neuron: (x_1 x_2 x_6)
- first layer second neuron: (x_3 x_5 x_7)
- first layer third neuron: (x_4 ΔF)
- second layer, neuron: first layer neurons' outputs.

So one neuron in the first layer monitors absolute physical parameters of analyzable atyp-

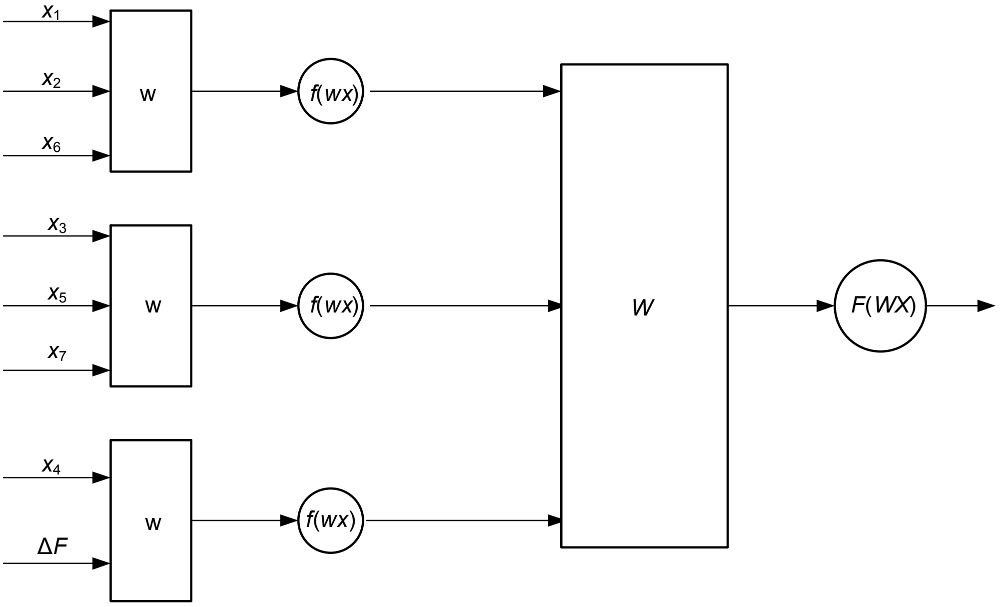

Fig. **Neural network diagram, regulating cargo points functioning** ical situation in cargo system functioning; the second neuron processes relative parameters, and the third monitors changes in economical estimation of every case both on the basis of growth (decline) of instant prime cost and the desired criterion deviation scope (4).

The second desired criterion which confirms operation (4) can determine correspondence cargo point functioning mode to its own optimal mode based upon the optimization of "point-buffer" system.

In case of exceeding some margin value of deviation neural network launches one of the several managing scripts.

The managing scripts each represent a solution of a management problem with the feedback adapted to supply control system.

Flow control via incoming buffer can be presented as one of the management options which is the most obvious operational experience on railway transport.

Cargo point functioning management script means applying a management that reduces current point state to the optimal calculated trajectory with a sufficient credibility value or a deviation not exceeding certain value.

Search methodology of optimal cargo point functioning mode is reduced to applying minimum expenses criterion to cargo system functioning (to accidental processes that are similar to inventory control systems)

J.I. Ryjikov offers to apply minimization principle of defined functional. We accept such interpretation of this system relative to cargo point that for cargo point functioning it is important not to allow any interruption of loading process. So it is important to synchronize cargo point functioning mode to distribution of rolling-stock supply intensity.

Let optimal performance or certain cargo point load be defined as S value. Freight flow expenses per time unit per area unit (buffer capacity) defined as h, and conditional penalty value (which can be estimated as less received income of cargo point exploitation) at a time unit per area unit (buffer capacity) arranged as d value.

Optimization of cargo point functioning expenses are accumulated in value c per area unit (buffer capacity) at a time unit, and cargo point freight flow defined as .

Let's pay attention to the fact that these volumes are not dynamically examined and all the integration and differentiation would be performed by the freight flow value.

Then the functional of total expenses on creation and maintenance of every cargo point stable functioning mode depending in the morphological sense on process technology, equipment, properties of each cargo point can be defined as:

$$+ d\int_{S}^{+\infty}(q - S)f(q)dq + c(S - z), \qquad (6)$$

where z - residual level of freight flow at the end of research period.

Then, according to formula (2):

$$\frac{dL}{dS} = \int_{0}^{S} f(q)dq - d\int_{S}^{+\infty} f(q)dq + c. \qquad (7)$$

If we apply methods considered in [5] to expression (7) we get

$$\frac{dL}{dS} = \int_{0}^{S} f(q)dq - d\int_{S}^{+\infty} f(q)dq + c =$$

$$= hF(S) - d(1 - F(S)) + c. \qquad (8)$$

Equating (8) to zero and expressing freight flow distribution function of time as a probability distribution function we get:

$$F(S) = \frac{d - c}{d + h}. \qquad (9)$$

Here $F(S)$ - probability distribution function over a research period.

This function's specific form is determined by the incoming freight flow character (or rolling-stock). It is possible to apply both of these conditions.

However, freight flow coming through the system is a function of time, therefore it is necessary to apply dynamical problem of freight flow management to the resulting solution.

Then cargo point is represented as a logical-and-dynamic system with feedback and a regulator of neural network form. The logical part of analyzing the system will be presented in the form of the referred above neural network and the dynamic one is expressed as differential equations.

Cargo point represents a dynamic system which can be expressed as evolutionary differential equations in terms of management theory.

Let cargo point freight flow variation be assigned as an equation

$$g(t) = b_n \frac{d^n Q}{dt^n} + b_{n-1} \frac{d^{n-1} Q}{dt^{n-1}} + \ldots + b_0 Q(t), \quad (10)$$

where $Q(t)$ - freight flow throughout cargo point;

$b_n, \ldots, b_0$ - derivative coefficients of corresponding degrees; $g(t)$ - forcing function that can be determined both on the basis of external statistics and prediction data.

As a whole, coefficient values of variable-order differential can be defined using mathematical apparatus of regression analysis. For the function value we assume that the first-order derivative is determined by the first row differences data, the second-order derivative is determined by the second row differences data etc.

The most frequent solution is evolutionary equation in the form of:

$$g(t) = b_2 \frac{d^2 Q}{dt^2} + b_1 \frac{dQ}{dt} + b_0 Q(t) \quad (11)$$

or

$$g(t) = b_2 \frac{d^2 Q}{dt^2} + b_0 Q(t). \quad (12)$$

Characteristic equations are accordingly presented as:

$$0 = b_2 p^2 + b_1 p + b_0 \quad (13)$$

and

$$0 = b_2 p^2 + b_0. \quad (14)$$

Thus, for a certain cargo system (point) freight flow dependence on time is formulated as a result of differential equation [4] solution $Q(t)$.

This information is compared to the calculated optimal cargo point load and put into one of the inlet neurons of the network.

While solving the task of cargo point managing it is possible to set out a number of problems besides the problem of minimum expenses of freight flow processing [2].

Managing parameters in this task can belong to a certain closed set. To solve such problems management quality functionals are introduced.

If cargo point develops as a dynamic system then it inevitably comes a point when external interference in point functioning is necessary.

Thus, process beginning moment is based on external reasons such as t_0, and influence ending moment is determined by the first moment of cargo system $(t, Q(t))$ achievement of some given surface of possible values Γ. The dimension of this surface is a unity more than the dimension of state system space.

In respect to practice and operational functioning a class of permissible processes can be assigned $Д(t_0, Q_0, u(t))$, which includes management ending moment t_1, trajectory of cargo point development $Q(t)$, management $u(\cdot)$, satisfying equation (2) and starting task condition $Q(t_0) = Q_0$.

The determination of management quality functional and its solution using the principle of maximum is described.

Certain management expression depends on the sort of disturbing effects on cargo system and current dynamics of freight flow during the period in question.

Management quality functional for the purpose of cargo system management tasks can be presented as

$$J_0 = \int_{t_0}^{T} F_0(t, Q(t), u)\, dt, \quad (15)$$

here T - management ending moment, which can be preset or determined by certain process trajectory; F_0 - scalar defined function; u - management.

Applying the described above principle of optimal cargo point load depending on the amount of freight flow processing portion allows to use this value as a minimizing functional

$$L = dq \times f(Q(t)) - cz \times f(Q(t)) -$$
$$-(d+h) \iint_J u(q, Q(t)) Q(t) f(q)\, dQ\, dq, \quad (16)$$

where domain J - class containing multiple of time moments, freight flow functions of time and freight flow portion amount functions of time; $f(Q(t))$ - function correcting freight flow influence.

In one-dimensional case (without freight flow throughout cargo point correction) the defined expression will be presented as

$$L = dq - cz - (d+h)\int_{0}^{S} u(q)q(t)f(q)dq. \quad (17)$$

Regulated cargo system development modeling requires the elaboration of function effectiveness criteria and determines the concept of “effective management”. These concepts are connected with economical effect analysis.

A.M. Gadjinsky notes that the economical effect of applying logistic method to freight flow management in production and circulation sphere is considered in the context of the following items:

- reserve reduction throughout all the movement of freight flow;
- cargo passing time reduction within logistic row;
- decreasing transport expenses;
- lowering expenses of hand work and corresponding expenses on cargo processing.

In conditions of examined railway cargo systems functioning these items are represented in a slightly different way:

- reducing buffer capacity of logistic system to optimal limit;
- decreasing of transport system functioning expenses;
- cargo system functioning mechanization.

Cargo system functioning effectiveness can be defined as

$$E = \frac{R}{Z}, \quad (18)$$

here R - economical effect of changing cargo system functioning; Z - expenses that are connected with management changes of cargo point functioning.

Management changing expenses can be described as

$$Z = C + nK,$$

where C - operating costs; K - capital investments; N - reduction coefficient.

Considering the system in a dynamic way we get:

(19)

where $\overline{Z}$, $\overline{R}$ - average values of expenses and result of introduction of a new cargo point management system accordingly.

Expenses effectiveness flexibility is determined this way. Considering this proportion effectiveness index sensitivity to the value of economical effect can be defined.

However, both economical effect and changing cargo point functioning process expenses can be dependent on time and freight flow passing throughout point:

$$E = \frac{R(t,Q)}{Z(t,Q)}. \quad (20)$$

Value of introduction of a new cargo point management system effect can be presented as:

$$E = \frac{R(t,Q)}{Z(t,Q)} = \quad (21)$$

$$= \frac{R_{лок}(t,Q) + R_{мз}(t - t_{техн}, Q) + R_{u}(t - \tau, Q)}{Z(t,Q)}.$$

In formula (22) $Z(t,Q)$ - expenses on freight flow exploration $Q(t)$.

Three items of economical effect are provided here:

1. Local level economical effect $R_{лок}(t,Q)$ appears on the cargo point level (cargo system)
2. Mezolevel economical effect $R_{мз}(t - t_{техн}, Q)$ appears after some lag $t_{техн}$;
3. Global level economical effect (relative to chosen system limits) $R_{u}(t - \tau, Q)$ appears in a typical system reaction time τ.

However, effectiveness of investing in a new cargo point management system can be examined from all the logistic process participants’ positions. In this case fraction’s numerator and denominator are represented as algebraic sum of vector effects and expenses of client and transporter.

In these conditions effectiveness term can be described as full function differential:

(22)

Composed partial second-order derivative of effectiveness is the direction changing index of the synergy effect value of expenses and economical effect modification as a result of introducing new cargo point management system.

Cargo point management system conversions can be considered as the original railway transport investment project applying some estimation indexes of cost-effectiveness.

Besides standard estimation indexes of cost-effectiveness (such as discount net profit, standard and discount pay-back periods, internal norm of project revenue performance) it is necessary to apply some integral indexes coordinating modified value of freight flow throughout cargo point $Q(t)$ with amount of currency flows.

Sometimes these currency flows are not migration of capital just as it is and perform some conventional role. These indexes mathematically evaluated as integral functional.

Difficulty of expressing such functional can be also explained by the fact that besides formal expression of any cargo point process functioning modification effects it is necessary to take into account modification influences occurred in functioning effectiveness of adjacent railway transport services with a set of cooperating each other effects.

Thus, resulting managing cargo point functioning system is the synthesis of knowledge acquired in management theory, theory of stochastic processes, logic and foundations of neural network constructing.

Such methodology is a pilot project for the railway transport, but it can be successfully applied to complex transport-logistic terminal. Its prospects conclude in building of logistic cargo flow plan in the towns and cities of our country because it allows any object adaptation and additional modules integration (controlling and market reaction measuring systems). Thus, city system considering, for example, should contain module taking into account population behavior and its spatial motion on city territory in time, therefore it is necessary to apply macro system theory.

Furthermore current methodology can be technically realized using existing hardware and software and prospectively construct an hierarchic computer system unit of transport activity automatic regulation, social and medical institutes, emergency services.

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