ON ECONOMIC AND MATHEMATICAL SUPPORT TO DECISION MAKING ON INVESTMENT PROJECTS UNDER UNCERTAINTY

© 2008 A.I. Chegodaev*

Key words: decision making, uncertainty, investment projects, investment efficiency index, mathematical methods, matrix game, solution of game, profitability, game against nature, rate of return.

New decision making methodology on investment projects (under uncertainty) is being discussed. This methodology is based on ideas of using: 1) more than two investment efficiency indexes; 2) interrelations of investment efficiency indexes, 3) mathematical methods of matrix game theory, linear programming, the theories of probability, mathematical statistics, numerical techniques and computers, when making a choice of optimal investment projects.

The problem of decision making on investment projects under uncertainty is actual for all enterprises connected in a varying degree with investment activity. Capital investment project evaluation through the utilization of four investment efficiency indexes is widely used in world practice. In order to increase the reliability choosing the investment project variant, firms focus not only on one, but two investment efficiency indexes, using the first as the main measure, and the second as the support investment efficiency index[1].

This research represents a statement of new methodology of decision making on investment projects, based on ideas of using: 1) more than two investment efficiency indexes, 2) interrelations of investment efficiency indexes, 3) mathematical methods of matrix game theory, linear programming, the theories of probability, mathematical statistics, numerical techniques and computers, when making a choice of optimal investment projects.

Investment processes may be an object of quantitative financial analysis. From the financial point of view they are combined by two processes: 1) creating an outfit or another object (capital accumulation), 2) successive acquisition of income. Capital accumulation and acquisition of income run successively (with or without a gap between them) or simultaneously in a certain period of time. In the last case it is supposed that investment returns starts before the investing process is finished. The two mentioned processes may have different in time distributions of payoff flows. The payoff flows, characterized as a single succession, are the object of the analysis. Referring to production investment, in most cases these flows are formed from investment expenditure and net income indexes.

The analysis of production investment generally consists in valuation and comparison of alternative investment projects effectiveness. As measurements here are used both formal characteristics, based on estimated income and expenditure flows discounting, and indexes, which are determinated on accounting data.

No matter what kind of investment evaluation method is chosen, in either event it is connected with presentation of investment expenditures and investment expenditures incomes by one and the same point of time. Most significant here is the choice of interest rate level, according to which the discounting is made. Risk is one of the main problems while comparing and choosing variants of investment.

Four indexes are generally used in financial analysis of investment efficiency: net adjusted income, payoff period (n_{ok}), internal rate of return (IRR), profitability (U).

The influence of investment expenditure and income on *NVP* can be represented as follows:

$$NPV = \sum_{j=1}^{n_2} E_j V^{j+n_1} - \sum_{t=1}^{n_1} K_t V^t, \qquad (1)$$

where K_t - investment expenditures in the period t; E_j - income in the period j; n_1 - investment; n_2 - investment returns process time; V - present value factor at the rate q (rate of collation).

In formula (1) it is assumed that the process of returns goes right after the investment is over, and the index *NVP* is ascertained by

* Chegodaev Anatoliy Ivanovich, Cand.Sc. Physics and Mathematics, Associate Professor of Yaroslavl Antiaircraft Missile Military College of Air Defence (Military Institute).

adjustment of appropriate sums to the beginning of the investment process. At the same time the valuation *NVP* is possible and practically important at the point of investing accomplishment or another point of time *t*. Then

$$(NPV)_t = (NPV)_0 (1+q)^t, \qquad (2)$$

where $(NPV)_0$ and $(NPV)_t$ are dimensions of net adjusted income, meant for the beginning of investing process and a period of time *t* after, q - rate of collation.

It is clear that while comparing some projects the moment of valuation should be common for all them.

Payoff period stands for the period when the sum of net adjusted income, discounted by the moment of investing accomplishment, is equal to the investment sum.

If there is no regularity in income distribution, and K is the investment rate set to the beginning of returns process time, then payoff period will be ascertained by summation of successive terms of income range, discounted at the rate q till the sum equal to volume of investments is realized. If the income is yielded at year-end, the sum is ascertained

$$S_m = \sum_{t=1}^{m} R_t V^t,$$

where - investment income for a point of time *t*, and .

Payoff period is equal *m* plus a certain part of the year which approximately makes

$$\frac{K - S_m}{R_{m+1} V^{m+1}}.$$

If investment incomes are equal and uniformly arrive once at year-end then payoff period is figured out from the formula

$$n_{ок} = \frac{\ln\left(1 - \frac{K}{R} q\right)}{\ln(1+q)}. \qquad (3)$$

The income rate q_b is meant by the internal rate of return (IRR), when $NPV = 0$,

$$f(q) = \sum_{t=1}^{t} \frac{R_t}{(1+q)^t} - I_0 = 0, \qquad \sum R_t V^t = 0,$$

where I_0 - investment spending which is supposed to be realized at a time at the moment of the project start date, R_t - annual *t*, returns, t - time from the beginning of the investment process, V - present value factor at the rate q_b, R_t - a term of payment flows, which can be a positive and negative quantity, t - time from the beginning of the investment process, q_b - solution of equation.

It should be noted that an approximate method is usually used to find the income rate q_b, particularly the method of successive iterations using tabulated value of discount coefficients. Two values of discount coefficient $q_1 < q_2$ are chosen for this with the use of charts thereby, in the range (q_1, q_2) the function $NPV = f(q)$ transforms its value from "+" to "-"and from "-"to "+". Then the formula is used

$$IRR = q_1 + \frac{f(q_1)}{f(q_1) - f(q_2)}(q_2 - q_1),$$

where q_1 - meaning of tabulated discount coefficients, when $f(q_1) > 0$ $(f(q_1) < 0)$; q_2 - meaning of tabulated discount coefficients, when $f(q_2) < 0$ $(f(q_2) > 0)$.

Calculation accuracy is inversely proportional to interval length (q_1, q_2), while the best approximation with the use of tabulated values is achieved when the interval length is short enough (e.g. it is equal $1\% = 0{,}01$).

The profitability U means the ratio of investment incomes and expenditures given to the same date:

$$U = \frac{R_j}{(1+q)^j} : \sum \frac{K_t}{(1+q)^t} \qquad (4)$$

where R_j - net income indexes, K_t - dimensions of investment costs $t = 1,2,\ldots,n_1$; $j = 1,2,\ldots,n_2$.

If the payment flow is made at the end of periods and temporary intervals between them are equal (payment flow is a permanent annuity in arrears) then profitability index is figured out of the formula

$$U = R\frac{a_{n;q}}{K} \qquad (5)$$

where $a_{n;q}$ - rent present value index, equal to $\frac{1-(1+q)^{-n}}{q}$, K — volume of investments, n - rent time-limit, time of investment return from the beginning of the rent first period till the last period.

The reliability of investment effectiveness quantitative analysis will be higher if we focus not on one but on two and more indexes of effectiveness when we choose the optimal investment variations.

It is possible if we take into account both the fact that the estimate of four effectiveness indexes of investment is based on reduction of non-contemporaneous outgoings to one and the same time and that the effectiveness indices are interconnected. Relatively simple connections between the effectiveness indexes may be settled in case the inpayments are distributed even and discrete and the cash flow may be represented by one or another coefficient financial rent.

The relation "profitability - payoff period" is formulated

$$U = \frac{1-(1+q)^{-n}}{1-(1+q)^{-n_{ок.}}}. \qquad (6)$$

The relation "profitability - internal rate of return" can be represented as follows:

$$U = \frac{q_b}{q} \cdot \frac{1-(1+q)^{-n}}{1-(1+q_b)^{-n}}. \qquad (7)$$

Formula

$$n_{ок.} = -\frac{\ln\left\{1-\frac{q}{q_b}\left[1-(1+q_b)^{-n}\right]\right\}}{\ln(1+q)} \qquad (8)$$

correlates "payoff period - internal rate of return".

Let us bring in consideration matrix game against "nature", where investor is the first player, whose pure strategies are different variants of investment projects, and as the second player is "nature", whose pure strategies are the following effectiveness indexes: internal rate of return q_b, profitability U and payoff period $n_{ок}$. As is known matrix game entirely is given by its matrix $H_{m\times n} = (h_{ij})$, $i = 1,2,...,m;\ j = 1,2,...,n$. In our case this matrix measures $K \cdot 3$, where K - number of rows (number of investment projects), number of columns in matrix (number of effectiveness indexes). All the elements of this matrix should be stated in the terms of the same "utility". It is possible to realize with the use of formulas (7), (8) and state the elements h_{ij} appropriate to profitability and payoff period indexes in terms of internal rate of return "utility" . Then the game against "nature" is decided in pure strategies. If this is a game with saddle point, then the investor chooses the number which matches the number of the row with saddle element of matrix $H_{K\cdot 3}$, in the quality of the optimal investment project.

In case the matrix game against "nature" doesn't have solution in pure strategies, the game should be solved in mixed strategies. Keeping in mind such end in view, it is necessary to simplify the game minimizing the number of rows of matrix $H_{K\cdot 3}$ through the properties of matrix game and saving the number of its columns, and then summarize the solution of this game with another matrix $H'_{K'\cdot 3}$ for linear programming problem. If $K' = 3$ and investor's optimal mixed strategy $\bar{x}^* = (x_1^*, x_2^*, x_3^*)$ is found where x_i^* - probability of investor's selection of investment project $i, i = 1,2,3$, then investor will use simultaneously three projects respectively with probabilities x_1^*, x_2^*, x_3^* which meet conditions $x_1^* + x_2^* + x_3^* = 1$. Then the problem, which is dual to the considered linear programming one, should be solved and the optimal mixed strategy of "nature" should be found, that means to figure out the probabilities y_1^*, y_2^*, y_3^*, where $y_1^* + y_2^* + y_3^* = 1$ should be used with effectiveness indexes to achieve maximal benefits. Payoff function $H(x,y) = \sum_{i=1}^{3}\sum_{j=1}^{3} h'_{ij} x_j y_j$ permits to find the maximal income $H(x^*, y^*) = \sum_{i=1}^{3}\sum_{j=1}^{3} h_i^* x_i^* y_j^*$ stated in terms of internal rate of return "utility". Finally, to increase the reliability while choosing the optimal investment projects investor can focus on four effectiveness indexes (payoff period, internal rate of return, profitability, net adjusted income). If make use of one more formula which states the relation "- internal rate of return":

$$(NPV)_n = (NPV)_0 (1+q_b)^n, \qquad (9)$$

where $(NPV)_0$ - volume of net adjusted income, meant for the beginning of investment process at the discount rate equal to internal rate of return q_b, and $(NPV)_n$ - volume of net adjusted income at the point of time n, where n — overall period of investment returns.

The investor's objective resolves itself to decide the game against "nature" with matrix $H_{K\times4}$. This problem solution is similar to solution of the game against "nature" with matrix $H_{K\times3}$. Elements of matrix $H_{K\times4}$ can be stated in terms of internal rate of return "utility" with the use of formulas (7), (8), (9).

The choice of rate q_b thereto can be explained by the fact that internal rate of return is in investors favor.

From the economic point of view the advantage of using the mentioned four effectiveness indices for the final choice of optimal investment projects is obvious as the accounting of all the four indices influence is maximal.

And with it not only one but sereal best investment projects making the maximal profit are recommended for using if the mentioned projects are used with strictly defined probability calculated in advance.

The calculation must be made by the modern computers according to the proframs worked out beforehand.

These computer calculations are also necessary for inner norm of profitability q_b determination by approximate methods, in particular the method of successive iterations with the application of discounting factors' tabulation accounts.

Let's note that the results of evaluation indices of effectiveness are nominal as they depend on time factor and the indicated calculations are based on discounting of cash flow, using the parameters of prospective income, its rate and time of inpayments and the choice of percantage rate.

But these parameters can be changed by various factors, in particular, by market sale, price variation and demand for the production of formed production systems, political situation in the country and in the world, situation with monetary and currence marketplace, changes in economic conjuncture and so on... In connection with this the safety and efficiency in analysis must be so high that the received data could be fundamental for making investment decisions.

---

[1] See: *Van Horn G.K.* Foundations of Finance Running. M., 1996; *Lipsii V.V., Kossov V.V.* Investment Project: Methods of Preparation and Analysis. M., 1996; *Neiman G., Morgenstern O.* Theory oa Games and Economic Behaviour. M., 1970; *Diubin G.N., Suzdal V.G.* Introduction into Applied Theory of Games. M., 1981; *Chegodaev A.I.* Foundations of the Theory of Final Antagonistic Games and their applying to the Decision of Problems and of Economics and Soldiery: Jaroslavl, 1993; *Vagin S.G.* Basic Tendencies and Conditions of Development of Running Large Social-economical Systems // Samara. 2007. № 5 (31).