THE THEORY OF FUZZY SETS AND ITS APPLICATION TO THE PROBLEM OF DECISION-MAKING IN THE CONDITIONS OF FUZZY INFORMATION, FUZZY RESTRICTIONS AND AMBIGUITY

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Keywords: fuzzy set, problem of decision-making, fuzzy information, fuzzy restrictions, fuzzy objective, ambiguity, membership function, universal set, random set theory.

The concept of fuzzy set of L. Zadeh, operations with fuzzy sets, their properties are considered. The value of fuzzy sets for mathematical modeling of complex systems, for adequate display of reality; generalization and development of fuzzy sets; application of the theory of fuzzy sets to the problem of decision-making in conditions of fuzzy information, fuzzy restrictions, ambiguity by means of solving the three practical problems: 1) problem of choosing the workplace, 2) problem of achieving the fuzzy objective of Bellmann-Zadeh, 3) problem of the account of every possible combination of the objective and the restrictions is also considered

At present mathematical methods of description and analysis of complex economic, social, technical, ecological and other systems are applied extensively. However, the language of traditional classical mathematics, based on the theory of sets and bivalent logic, is not flexible enough for modeling really complex systems. The fuzziness of the world surrounding us is essentially reflected in mathematical models which do not possess the property of adequacy of display of the world, the reality and become unsuitable for practical application. Besides it is necessary to pay attention to another aspect of the fuzziness of the world which is inherent in thinking and perception of a person. For mathematical modeling of the real-life phenomena the language of the theory of fuzzy sets of Lofty A. Zadeh is more flexible and suitable.

L. Zadeh formulates the definition of fuzzy set as follows: a fuzzy subset À of the universal set U is characterized by the membership function $\mu_A : U \to [0,1]$, which associates each element $u \in U$ with the number $\mu_a(u)$ from segment $[0,1]$, describing a grade of membership of an element u to a subset À.

Crisp sets are special cases of fuzzy sets. For crisp sets $\mu_A(u) = 1$, if $u \in A$, or $\mu_A(u) = 0$, if $u \notin A$. The terms “fuzzy set” and “membership function” are used as synonyms.

Fuzzy subsets with membership function, accordingly, are referred to as meet $A \cap B$, product AB, union $A \cup B$, sum $A + B$, negation $\overline{A}$

$$\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u)),$$
$$\mu_{AB}(u) = \mu_A(u)\mu_B(u),$$
$$\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u)),$$
$$\mu_{A+B}(u) = \mu_A(u) + \mu_B(u) - \mu_A(u)\mu_B(u),$$
$$\mu_{\overline{A}}(u) = 1 - \mu_A(u),$$

where A and B are two fuzzy subsets of the universal set U with the membership function $\mu_A(u)$ and $\mu_B(u)$ accordingly, and $u \in U$.

The theory of fuzzy sets is the result of finding a solution to the various applied problems concerning the theory of algorithms, artificial intelligence, the theory of decision-making, the theory of recognition of images, etc. The theory of fuzzy sets is the unique theory which mathematically operates the semantics of speech. L.Zadeh has shown how fuzzy, qualitative character of information can be used in the formalized procedures of analysis.

The generalization of the device of fuzzy sets of L. Zadeh was offered by A.I.Orlov in 1975. A.I. Orlov suggested to consider the random set theory. It allows to include the theory of fuzziness in probability theory. Its application is useful in mathematical economy, expert estimations, modeling the patterns of forest fires, etc.

The problem of decision-making by means of the theory of fuzzy sets in conditions of fuzzy information and fuzzy restrictions is studied. Thus it is supposed, that X is a

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universal set of alternatives which is at the disposal of DMP (decision making person), and $B = \{b_1, b_2, ..., b_m\}$ is a set of conditions (restrictions) of performance of any operation, and $X = \{x_1, x_2, ..., x_m\}$ is the final set of alternatives. We shall apply the fuzzy set B_j constructed on the set of alternatives X, with the membership function $\mu_{B_j} : X \to [0,1]$ to the condition $b_j \left(j = 1,2,...,m\right)$. Fuzzy set will be called "fuzzy restriction". The higher the grade of membership of alternative to the fuzzy set of restrictions B (i.e. the more value), the more the degree of achievement of this restriction in case of choosing the alternative is.

Let's consider the problem illustrating decision-making in conditions of fuzzy restrictions: we have to choose one of the four places of work, each of which is estimated by N experts, independent and equal in rights; the set of restrictions B contains six criteria (attributes): $B = \{b_1, b_2, b_3, b_4, b_5, b_6\}$, where b_1 is an opportunity to be engaged in scientific work, b_2 is the prospect of growth, b_3 is material benefit, b_4 is good team, b_5 is convenient location, b_6 is prestige; the universal set is named $X = \{x_1, x_2, x_3, x_4\}$, where x_i is the workplace.

As a result of processing the given information the fuzzy subsets $B_j \left(j = 1,2,3,4,5,6\right)$ of the set X with the membership function $\mu_{B_j} : X \to [0,1]$ whose values are specified in table 1 are received, thus the fuzzy solution to the problem of decision-making with fuzzy restrictions is called meet D of the fuzzy sets of all restrictions, and the membership function of this meet is the following:

$$\mu_{D_i}(x_i) = \min\{\mu_{B_1}(x_i), \mu_{B_2}(x_i), \mu_{B_3}(x_i),$$
$$\mu_{B_4}(x_i), \mu_{B_5}(x_i), \mu_{B_6}(x_i)\}$$

Generally the way of solving such problem is called defuzzification and consists in the transition from fuzzy representations to the corresponding precise administrative decisions. One of the widespread ways of defuzzification is the following:.

$$\max_{x_i \in X} \mu_Д(x) =$$
$$= \max_{x_i \in X} \min\{\mu_{B_1}(x), \mu_{B_2}(x), ..., \mu_{B_m}(x)\},$$

In our case such alternative is x_3:

$$\max_{x_i \in X} \mu_Д(x) = \max_{x_i \in X} \min\{0;0,1;0,2;0\},$$

and $\mu_Д(x_3) = 0{,}2$.

Mathematical analysis of any problem requires considering the fuzzy objective and the set of fuzzy restrictions, namely, every possible combination of the objective and the restrictions.

To find the solution to a problem it is necessary to consider each of the six possible rearrangements which can be made with the three numbers 1, 2, 3, meaning accordingly: the number of the workplace and the number of the worker, agreed previously. For example, rearrangement (3, 1, 2) shall be interpreted as follows: the third worker is on the first workplace, the first worker is on the second workplace, and the second worker is on the third workplace. Such interpretation applies to the five remaining rearrangements: (1, 2, 3), (1, 3, 2), (3, 2, 1), (2, 3, 1), (2, 1, 3). The process of a finding the

Table 1

Place of work x_i	**Restrictions (attributes)**						$\mu_D(x)$
	b_1	b_2	b_3	b_4	b_5	b_6	
	Values of membership function						
	$\mu_{B_1}(x)$	$\mu_{B_2}(x)$	$\mu_{B_3}(x)$	$\mu_{B_4}(x)$	$\mu_{B_5}(x)$	$\mu_{B_6}(x)$	
x_1	0,1	0,8	0,3	0	0,2	0,2	0
x_2	0,4	0,7	0,1	0,6	0,9	0,9	0,1
x_3	0,3	0,2	0,8	0,5	0,4	0,4	0,2
x_4	0,5	0,4	0	0,4	0,8	0,8	0

Table 3

X	Conditions (restriction)			Objective	Decision
	μ_{11}	μ_{22}	μ_{33}	μ_G	$\mu_Д = \mu_{123}$
x_1	0,1	1	0,2	0,3	0,1
x_2	0,3	0,9	0,3	0,3	0,3
x_3	0,5	0,8	0,5	0,5	0,5
x_4	0,6	0,7	0,7	0,4	0,4
x_5	0,7	0,7	0,8	0,1	0,1

Table 4

X	μ_{123}	μ_{132}	μ_{312}	μ_{321}	μ_{231}	μ_{213}
x_1	0,1	0,1	0,3	0,3	0,3	0,2
x_2	0,3	0,3	0,3	0,3	0,3	0,3
x_3	0,5	0,5	0,4	0,4	0,5	0,5
x_4	0,4	0,4	0,4	0,4	0,4	0,4
x_5	0,1	0,1	0,1	0,1	0,1	0,1
max	0,5	0,5	0,4	0,4	0,5	0,5

solution to a problem consists in finding the most effective rearrangement among the six specified. Let's consider the example of rearrangement (1, 2, 3) (Table 3)

The results of the analysis of all the six rearrangements under the specified scheme are given in Table 4.

The results of the analysis are presented by last line of this table. So, the solution to the problem are the four rearrangements (1, 2, 3), (1, 3, 2), (2, 3, 1), (2, 1, 3) for which the estimation x_3 = (positive efficiency) has the greatest grade of membership*.

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Received for publication on 28.04.2009